LinAlg Recap Exercise

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1 LU decomposition

Consider the following matrix $A \in \mathbb{R}^{3\times 3}$ where $p \in \mathbb{R}$:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 2 & p & p \end{pmatrix}$$

1.1

Write down elimination matrices E_{21}, E_{31}, and E_{32} that introduce zeros in the $(2,1)$ $(3,1)$, and $(3,2)$ entries so that $E_{32}E_{31}E_{21}A = U$ is upper triangular. Their entries may depend on p.

$$E_{21} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \quad E_{31} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \quad E_{32} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

1.2

Write down the lower and upper triangular factors L and U that multiply to make $A = LU$. The triangular factors may depend on the parameter p.

$$L = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \qquad U = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

1.3

Why is A not invertible if $p = -2$?

1.4

If $p \neq -2$, $\quad \text{rref}(A) =$ $\begin{pmatrix} & \\ & \end{pmatrix}$

1.5

Let $p = -2$. Find $\text{rref}(A)$ and bases for $N(A)$ and $C(A)$.

1. $\text{rref}(A) =$ $\begin{pmatrix} & \\ & \end{pmatrix}$
2. A basis for $C(A)$ is given by: $\left\{ \begin{matrix} \\ \\ \end{matrix} \right\}$
3. A basis for $N(A)$ is given by: $\left\{ \begin{matrix} \\ \\ \end{matrix} \right\}$

2 References

Exercises 1.1−1.3: https://github.com/mitmath/1806/blob/master/exams/exam1.pdf