LinAlg Proof Exercises Week 12

Felix Breuer

December 2023

1

Let $T, S \in R^{n \times n}$ and S be invertible.

1. Prove that T and $S^{-1}TS$ have the same eigenvalues
2. What is the relationship between eigenvalues of T and those of $S^{-1}TS$?

2

Prove the following:

Hint: Start with $Av = \lambda v$.

1. $\lambda + 1$ is an eigenvalue of $A + I$
2. λ^{-1} is an eigenvalue of A^{-1}
3. λ^2 is an eigenvalue of A^2

3

Let $T \in R^{n \times n}$. Prove that if $Tv = 3w$ and $Tw = 3v$, 3 or -3 is an eigenvalue of T.

4

Let $u, v \in \mathbb{R}^2$. Show that u is an eigenvector of $Auv^T \in \mathbb{R}^{2 \times 2}$. Find both eigenvalues of A.

5 Sources

Exercises 1,3: https://github.com/mitmath/1806.
Exercises 2,4: S. Axler, Linear Algebra Done Right. https://link.springer.com/book/10.1007/978-3-319-11080-6.