LinAlg Recap weeks 1-7

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1 True/false and open questions

For the following questions let $x \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{m \times n}$ and V be a vector space.

1. Why can $||x||$ never be negative?
2. $||x|| = 0$ if and only if _____
3. If A is invertible, $\text{rank}(A) =$ _____
4. When is $U \subseteq V$ a subspace of V?
5. We can compute the $A = CR$ decomposition with the Gauss-Jordan algorithm (to compute $rref(A)$)
6. Consider B: The number of linearly independent rows does not always equal the number of linearly independent columns.
7. How would you prove a set of vectors $B \subseteq V$ is a basis of V?
8. If any vector $v \in \text{span}(v_1, \ldots, v_n)$ can be uniquely expressed as a linear combination of $v_1, \ldots v_n$, we call $v_1, \ldots, v_n$ _____
9. A basis for the set of polynomials with real coefficients of degree less than or equal to 3 is given by $\{ \quad \}$
10. Let $\mathbf{B}$ be a basis of V and $\mathbf{C}$ be a generating set of V ($\text{span}(\mathbf{C}) = V$). How do $\mathbf{B}$ and $\mathbf{C}$ differ?
11. Multiplying A with elimination matrices from the left doesn't change the span of rows and span of columns of A
12. If $\dim N(A) > 0$ we know that $Ax = b$ does not have a unique solution
13. How can you compute A^{-1} (assuming it exists)?
14. $C(B)$ is a subspace of $\mathbb{R}^n$
15. What can we say about A if $A^4 = I$? What kind of matrix could A be?
16. All bases of subspaces of V have the same number of vectors