LinAlg Quiz Week 13

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1 Quiz

Let $A, Q \in \mathbb{C}^{n\times n}$ and $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of A. The trace of A is defined as $\mathrm{Tr}(A) = \sum_{i=1}^{n}(A)_{ii}$, $A^* = \overline{A}^T$.

1. $\det(A^*) = \det(A)$
2. The eigenvalues of a triangular matrix are given by its _____ entries. (Note: A diagonal matrix is triangular)
3. $\sum_{i=1}^{n} \lambda_i = \mathrm{Tr}(A)$, $\prod_{i=1}^{n} \lambda_i = \det(A)$
4. Eigenvectors corresponding to different eigenvalues are not necessarily linearly independent
5. If we know all eigenvalues of A, we know if A is invertible.
6. What are the algebraic and geometric multiplicities of an eigenvalue λ?
7. By the fundamental theorem of algebra, any polynomial with real coefficients has real roots.
8. $\det(PA) = \det(A)$ when P is a permutation matrix (attained from swapping columns/rows of the identity matrix)
9. $\det(Q)| = \pm 1$ and $|\lambda| = 1$ if $Q \in \mathbb{C}^{n\times n}$ is an orthogonal matrix ($Q^TQ = I$) with eigenvalue λ. (Extra question: what if Q is unitary, i.e. $Q^*Q = I$?)