Final LinAlg Quiz Week 14

Felix Breuer

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1 Quiz

1. Let $a_1, \dots a_n$ be the columns of $A \in \mathbb{R}^{m \times n}$ and $b_1, ..., b_n$ be the rows of $B \in \mathbb{R}^{n \times p}$. Then $AB = \sum_{i=1}^{n} a_i b_i$
2. A is not invertible $\iff$ A has only eigenvalue 0
3. We can find the representation matrix A of a linear map $L : \mathbb{R}^n \to \mathbb{R}^m$ by taking $T(e_i)$ for all standard basis vectors e_i of R^n as the columns of A in any order.
4. A and $S^{-1}AS$ have the same eigenvalues.
5. Per the spectral theorem any symmetric matrix is diagonalizable with positive real eigenvalues.
6. What does the spectral theorem state about a symmetric matrix $A \in \mathbb{R}^{n \times n}$?
7. A matrix is invertible if and only if it is diagonalizable.
8. Give an example of a matrix that is not diagonalizable over $\mathbb{C}$.
9. Let $A = LDL^T$ where L is lower triangular and D diagonal with only positive entries along the diagonal. Prove that then A is positive definite.

   Hint: If $D = \mathrm{diag}(d_1, \dots d_n)$ and all diagonal entries are positive, $D = D^{1/2} D^{1/2}$ where $D^{1/2} = \mathrm{diag}(\sqrt{d_1}, \dots \sqrt{d_n})$